

## LTCC Basic Course

**Title:** Morse theory, Topology and Robotics.

**Basic Details:**

- Core Audience: Pure and Applied, 2<sup>nd</sup>/3<sup>rd</sup>yr
- Course Format: Extended: 5 x 2hr lectures

**Course Description:**

- Keywords: Morse functions, Morse inequalities, configuration spaces of mechanisms, mechanical linkages, Betti numbers of configuration spaces, topological problems inspired by robotics.
- Syllabus:

**Lecture 1.** Topology of sublevel sets, operation of attaching cells, Morse functions, Morse Lemma, existence of Morse functions, crossing a Morse critical value. Main theorem of Morse theory. Morse inequalities. Examples: spheres, complex projective spaces, etc.

**Lecture 2.** Smale's solution of Generalised Poincare Conjecture. H-cobordism theorem. Ideas and techniques of the proof.

**Lecture 3.** Planar linkages and their configuration spaces. Formula for their Betti numbers. Walker's Conjecture.

**Lecture 4.** Spaces of polygons in high dimensions, classification of these spaces in combinatorial terms.

**Lecture 5.** Robot motion planning and topology. Schwarz genus of a fibration. Lusternik-Schnirelmann theory. Topological complexity.

- Recommended reading:
  - o J. Milnor, Morse theory. 1963
  - o J. Milnor, Lectures on the h-cobordism theorem. 1965.
  - o M. Farber, Invitation to topological robotics, EMS, 2008.
- Additional Optional reading:
  - o M. Farber and V. Fromm, The topology of spaces of polygons. Trans. Amer. Math. Soc. 365 (2013), no. 6, 3097–3114.
- Prerequisites: Basic homology theory of cell complexes, basic knowledge smooth manifolds.

**Format:**

- No of discussion/problem sheets: 4

- Electronic lecture notes: yes

**Lecturer Details:**

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