Methods of Industrial Mathematics – Problem Sheet on Session 1.

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1. Using the Euler equations show that when $\omega_3 \gg both\ \omega_1 and\ \ \omega_2$ a rigid body spacecraft spins around its 3-axis at approximately a constant rate in absence of applied torques. Show that stability depends on the moments of inertia and that $\omega_1, and\ \ \omega_2$ either grow (resulting in instability) or oscillate. In the latter case determine the frequency of the oscillation. This oscillation is called nutation and is generally undesirable and eliminated by dampers.

2. Write down the equations for the three-body problem with gravity the only force.

3. Writing

$$g_1 = r_1 - l_1 - p_1 - s_1,$$
$$g_2 = r_2 - l_2 - d_2 - s_2,$$

solve the manpower equations analytically if initially at t=0 there are X_{10} workers and X_{20} managers. Non-trivial constant coefficient problems like this can offer one useful check on more elaborate problems.

4. Show that the third order Runge-Kutta scheme to solve

$$\frac{dy}{dx} = f(x, y);\ \ y(0) = y_0$$

Namely

$$x_{n+1} = x_n + h$$
$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 4k_2 + k_3),$$

where

$$k_1 = f(x_n, y_n)$$
$$k_2 = f\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1\right)$$
$$k_3 = f(x_n + h, y_n - hk_1 + 2hk_2)$$

is indeed third-order accurate with error $O(h^4)$. [Hint: use Taylor Series]. Third-order schemes seem not to be used so often in practice, but can in principle be. This exercise shows the method how the accuracy of a Runge-Kutta model model is proved with a bit less algebra than for the 4th order case.

5. **Challenge Question**. In the traffic model, model the second and subsequent cars and consider how the model might be made more realistic.