Methods of Industrial Mathematics – Problem Sheet on Session 3.

Issue No: 1.0

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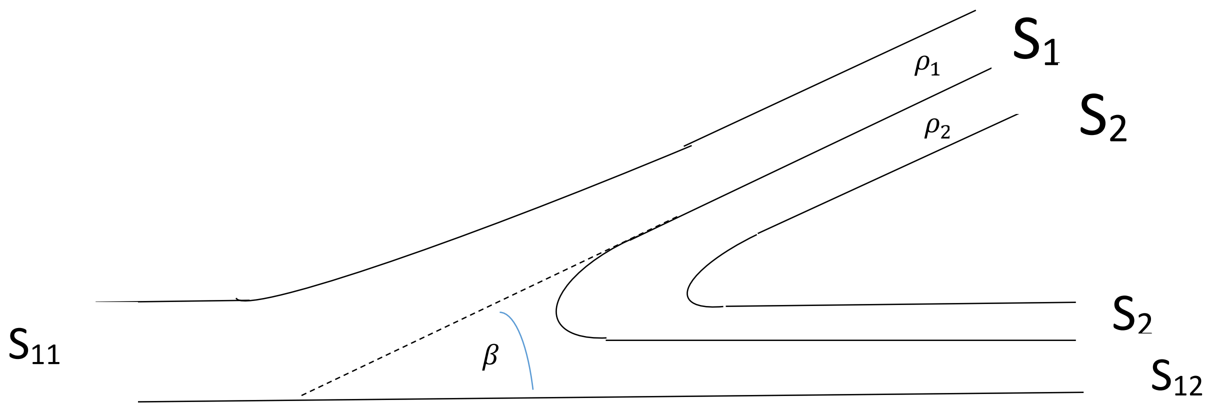
Date: 05/02/2018

1. Consider the steady-state incompressible viscous flow in an annular tube, centred on the z-axis, of inner radius $a$and outer radius b, where there is a constant pressure gradient $-G$ along the tube. Assume no-slip at the inner and outer tube walls. Assume no swirl. Show that the axial velocity component is

$$u_z = \frac{G}{4\mu}\left[-r^2 + \frac{(b^2 - a^2)}{ln\left(\frac{b}{a}\right)}\ln r + \frac{(a^2 \ln b - b^2 \ln a)}{ln\left(\frac{b}{a}\right)}\right]$$

2. A 2-D planar jet comprising two layers of inviscid incompressible fluids of densities ρ_1 and ρ_2 impacts a rigid frictionless wall at angle β as shown. Body force is negligible.



The cross-sectional areas per unit depth of the incoming jet layers are S_1, S_2. The first layer is divided in two; the portion with the free surface having cross-sectional area per unit depth S_{11}, while the portion adjacent to the second layer has cross-sectional area per unit depth S_{12}. If we put $S_{11} = \alpha S_1$ show that

$$\alpha = \frac{(S_1\rho_1 + S_2\rho_2)}{S_1\rho_1} cos^2\left(\frac{\beta}{2}\right)$$

What is happening when $\alpha = 1$? What is the force per unit depth on the wall?

3. Consider a uniformly stretching (linear velocity gradient) axisymmetric jet of density ρ with tip speed V_0 and tail speed V_L of initial length L_0 and radius R_0, with $V_0 > V_L$. Assuming inviscid, incompressible flow and zero pressure on the curved surface of the jet show that the pressure inside the jet is given by

$$p = p(r,t) = \frac{3\rho V^2}{8(L_0+Vt)^2}\left\{R_0^2\left(1+\frac{Vt}{L_0}\right)^{-1} - r^2\right\},$$

where $V = V_0 - V_L$.

4. Consider a shock in an initially stationary material at zero pressure and internal energy. Use the Rankine-Hugoniot equations with notation as in the slides to derive the following equations:

$$p_1 = \rho_0 U u_1,$$

$$p_1 = \frac{\rho_0 \rho_1 u_1^2}{(\rho_1 - \rho_0)},$$

$$e_1 = \frac{1}{2}u_1^2.$$

[This third result is due to Alt'Schuler and I think is rather striking and even elegant – half the energy of the shock goes into the internal energy and half into kinetic energy.]

5. **Challenge Question**. Consider a stretching jet initially of length L_0 of density ρ_J with tip speed V_0 and tail speed V_L with a uniform velocity gradient from front to back. Suppose the tip is initially at distance S, the 'stand-off' from the front face of a target of density ρ_T. Using the hydrodynamic penetration law of Hill Mott and Pack (see slide 62) calculate the total penetration of the jet in terms of S, L_0, V_0, V_L, and $\beta = \left(\frac{\rho_J}{\rho_T}\right)^{\frac{1}{2}}$. [Hints: You may find it helpful to introduce an initial co-ordinate q varying from zero at the jet tip to L_0; then consider the penetration as a function of q and find the time a jet element initially at q arrives at the bottom of the crater. Consider too how the length of a jet element is changing. Finally, there is no need to evaluate the pressure within the jet as in question 3 above – this is considered negligible compared with the huge pressure associated with the impacting jet.]