The classical moment problem

Mondays November 13th to December 11th, 10:50–12:50

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We shall discuss several topics related to the classical moment problem, which studies the relation between a measure μ on the real line and its sequence of moments

$$s_k = \int x^k d\mu(x) \ , \quad k = 0, 1, 2, \cdots$$

The two basic questions are whether a sequence (s_k) can be realised as the sequence of moments of a measure (existence), and whether such a measure is unique (determinacy); these turn out to be related to a variety of questions in classical analysis, functional analysis and operator theory.

1. Extension of positive functionals: the Riesz extension theorem
2. Quasianalyticity: the Denjoy–Carleman theorem
3. Orthogonal polynomials: quadrature formulæ, Chebyshev – Markov – Stieltjes inequalities
4. Jacobi matrices: spectral theory, Weyl circles

Familiarity with the Lebesgue and Lebesgue–Stieltjes integral (incl. the Riesz representation theorem) will be helpful.

References

[1] Akhiezer, N. I. The classical moment problem and some related questions in analysis. Translated by N. Kemmer Hafner Publishing Co., New York 1965 x+253 pp.

[2] Carleman, T. Les fonctions quasi analytiques. Gauthier-Villars, Paris, 1926