Measure Theory: Exercises 1

1. Consider the collection $\mathcal{A}$ of subsets $A_1, A_2, \dots$ of the integers such that $A_i = \{ni \mid n \text{ is an integer}\}$.
Determine what is $\sigma(\mathcal{A})$.

2. Assume that μ is defined on all subsets of X, that $\mu(\emptyset) = 0$, and for all increasing sequences $A_1 \subseteq A_2 \subseteq \dots$ it holds that $\lim_{i\to\infty} \mu(A_i) = \mu(\cup_{i=1}^{\infty} A_i)$. True or false: μ is finitely additive.

3. If $A_1, \dots, A_n$ are measurable sets each of finite measure show that $\mu(\cup_{i=1}^{n} A_i) = \sum_{S\subseteq\{1,2,\dots,n\}} (-1)^{|S|+1} \mu(\cap_{i\in S} A_i)$.

4. Determine the smallest sigma algebra on $\mathbf{R}$ that is generated by the collection of all one-point subsets of $\mathbf{R}$.