# Theory of Linear Models
# Exercises 3

27 January 2020

1. Consider the half-replicate fractional factorial design for three factors, each at two levels, plus two centre points:

| $X_1$ | $X_2$ | $X_3$ |
|---|---|---|
| -1 | -1 | -1 |
| -1 | 1 | 1 |
| 1 | -1 | 1 |
| 1 | 1 | -1 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

   Find a 95% confidence interval for $\beta_1 + \beta_{23}$ if $\widehat{\beta_1 + \beta_{23}} = 10.03$ and $s^2 = 5.34$. How would the result be interpreted?

2. Consider the following data:

| $Y$ | $X_1$ | $X_2$ | $X_3$ |
|---|---|---|---|
| 10 | -1 | -1 | -1.0001 |
| 12 | -1 | 1 | -0.9999 |
| 20 | 1 | -1 | 1.0000 |
| 22 | 1 | 1 | 1.0000 |
| 16 | 0 | 0 | 0.0000 |
| 18 | 0 | 0 | 0.0000 |

(a) Test the hypothesis $\beta_1 = \beta_2 = 0$ against a two-sided alternative.

(b) Test the hypothesis $\beta_1 = \beta_3 = 0$ against a two-sided alternative.

(c) Calculate the leverage of each observation.

(d) Assuming that the experiment was completely randomized, carry out a permutation test of $\beta_1 = 0$ against a two-sided alternative.

(e) Comment on the results.

3. For the general second order polynomial regression model, with $q$ explanatory variables, find the location of the stationary point as a function of the parameters. (Hint: rewrite the model using the vector $\mathbf{b}' = [\beta_1 \ \cdots \ \beta_q]$ and the matrix

$$\mathbf{B} = \begin{bmatrix} \beta_{11} & \frac{1}{2}\beta_{12} & \cdots & \frac{1}{2}\beta_{1q} \\ \frac{1}{2}\beta_{12} & \beta_{22} & & \vdots \\ \vdots & & \ddots & \frac{1}{2}\beta_{(q-1)q} \\ \frac{1}{2}\beta_{1q} & \cdots & \frac{1}{2}\beta_{(q-1)q} & \beta_{qq} \end{bmatrix}$$

and then use vector differentiation.)