

Theory of Linear Models
Exam Question
Mock 2026

April 2025

An experiment yielded the following data on a response variable Y and explanatory variables X_1 , X_2 and X_3 :

X_1	X_2	X_3	Y
-1	-1	-1	12.3
-1	-1	1	19.7
-1	1	-1	14.3
-1	1	1	13.9
1	-1	-1	4.6
1	-1	1	5.7
1	1	-1	2.2
1	1	1	8.1
0	0	0	19.4
0	0	0	1.5
0	0	0	1.1
0	0	0	19.7

The proposed model is the second order response surface model (including a constant) with Y_i 's uncorrelated and $Var[Y_i] = \sigma^2$.

1. Express the proposed model in matrix form, i.e. write down the model matrix $\mathbf{X}$ and the vector of parameters $\boldsymbol{\beta}$.
2. Find the rank of the matrix $\mathbf{X}$.
3. Find a generalized inverse of $\mathbf{X}'\mathbf{X}$.
4. Find a least squares estimator $\hat{\boldsymbol{\beta}}$ of $\boldsymbol{\beta}$. (Hint: Use the generalized inverse of $\mathbf{X}'\mathbf{X}$.)

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5. Show that β_{33} is not estimable.
6. Show that $\theta = \beta_{11} + \beta_{22} + \beta_{33}$ is estimable and find the form of its least squares estimator $\hat{\theta}$ and its observed value.
7. Find the variance of $\hat{\theta}$.
8. Write down the form of a $100(1-\alpha)\%$ confidence interval for θ . You are not required to calculate this interval.