Fundamental Theory of Statistical Inference

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What is statistical inference?

Observational data modelled as observed values of random variables, to provide framework from which inductive conclusions may be drawn about mechanism giving rise to data.

To analyse observations $y = (y_1, \ldots, y_n)$.

- Regard y as observed value of random variable $Y = (Y_1, \ldots, Y_n)$ having an (unknown) probability distribution specified by a probability density function, or probability mass function, $f(y)$.
- Restrict the unknown density to a suitable family $\mathcal{F}$, of known analytical form, involving a finite number of real unknown parameters $\theta = (\theta^1, \ldots, \theta^d)^T$. The region $\Omega_\theta \subset \mathbb{R}^d$ of possible values of θ is called the parameter space. To indicate dependency of the density on θ write $f(y; \theta)$, the 'model function'.
- Assume that the objective of the analysis is to assessing some aspect of θ, for example the value of a single component θ^i.

Types of inference

- Point estimation
- Confidence set estimation
- Hypothesis testing

Three paradigms of inference

- Bayesian
- Fisherian
- frequentist

Differences relate to interpretation of probability and objectives of statistical inference.

Bayesian inference (Bayes, Laplace)

Unknown parameter θ treated as random variable.

Key: specification of prior distribution on θ, before data analysis.

Objective or subjective specification. Inference is formalization of how prior changes, to posterior distribution, in light of data y, via Bayes' formula.

Use of probability distributions as expressing opinion.

Fisherian inference (Fisher)

Development of logic of inductive inference, releasing from a priori assumptions of Bayesian school.

Repeated sampling principle: inference from y founded on comparison with datasets from hypothetical repetitions of experiment generating y, under exactly same conditions.

Key elements of Fisherian statistics

- Central role played by likelihood, maximum likelihood.
- To be relevant to y, inference carried out conditional on everything known, uninformative about θ.

Frequentist inference (Pearson, Wald, Lehmann...)

Origins in detailed analysis of concepts developed by Fisher, likelihood and sufficiency.

Formal incorporation of optimality criteria.

Key elements of frequentist statistics

- Inference procedures as decisions problems, rather than as summary of data.
- Clarity in mathematical formulation.
- Optimum inference procedures identified before y observed.
- Optimality defined explicitly in terms of repeated sampling principle.